# Multigap Effect on Phonon Structure in Tunneling Experiments*

A. Leger and J. Klein
*Groupe de Physique des Solides de l'Ecole Normale Supérieure, Laboratoire associé au Centre National de la Recherche Scientifique, Faculté des Sciences, Tour 23, Paris V, France*
(Received 5 June 1970)

The $\vec{k}$ dependence of the energy-gap function leads to a double gap edge in tunneling characteristics of superconducting thick Pb. Nevertheless, it is shown here experimentally and theoretically that there is no corresponding splitting of the phonon peaks.

In a recent paper Blackford and March[1] have reported tunneling experiments between superconducting thin Pb films and Pb single crystals in which they observed multigap effects. In a given crystallographic direction, they clearly observed two values of the gap edge: $\Delta_1$ and $\Delta_2$. However, there was no evidence for any corresponding splitting in the phonon structure. Within a simple model, one would have expected two peaks in the tunneling curves (at energies $\hbar\omega_{ph}+\Delta_{1,2}$) for each phonon critical point ($\omega_{ph}$), but these, nevertheless, were not observed in experimental curves.

In their experiment, the energy gap of the Pb injecting electrode ($\Delta_F$) was very close to $\Delta_2$, and Blackford and March suggested that there was such an enhancement of the phonon structure associated with[2] $\Delta_2 = \Delta_F$ that the other one was hidden.

Here we report a similar experiment but where this last ambiguity is avoided by the use of a different injecting electrode. We use Al-Pb film junctions, the two metals being superconductors. Both films are annealed and the Pb films are 5 $\mu$ thick. Tomasch[3] has shown that such Pb films consist of oriented polycrystals with their [111] axis normal to the substrate. These junctions display two well-defined energy-gap edges $\Delta_1 = 1.25 \pm 0.01$ meV and $\Delta_2 = 1.40 \pm 0.01$ meV on $dV/dI$ characteristics [Fig. 1(a)]. The second derivative $d^2V/dI^2$ exhibits Tomasch resonances (labeled $t_n$) and a phonon structure quite similar to that reported by Blackford and March[1] [Fig. 1(b)].

The peak mentioned as $\omega_1$ (not well defined in thin-film junctions) is also present, but again there is no evidence for splitting in the phonon structure. No gap coincidence can be invoked here. From the experimental point of view, our resolution would have been sufficient to observe the expected splitting (for instance, in the longitudinal phonon peak) if it exists.

Let us now examine, theoretically, if such a splitting of the phonon structure is really to be expected in those experiments. Bennett[4] has proposed a theory for multigap superconductors and has shown that the anisotropy of the gap arises from the anisotropy of the phonon density of states involved in the Eliashberg equations. The effective tunneling density of states is given by[4]

$$\frac{N_T(E)}{N_0} = \int d\Omega g(\Omega)\,\mathrm{Re}\left(\frac{E}{[E^2-\Delta^2(\Omega,E)]^{1/2}}\right), \quad (1)$$

where $\Omega = (\theta, \phi)$ defines a general direction in the $\vec{k}$ space of the superconductor, and $g(\Omega)$ is the angular distribution (in $\vec{k}$ space) of tunneling quasiparticles. The simplest model (WKBJ approximation on quasifree electrons and specular transmission) would lead to a very anisotropic distribution, since most of the tunneling current comes from electrons with their wave vectors lying in a narrow cone (5° aperture) whose axis is normal to the barrier. But, the relaxation of $\vec{k}_{\parallel}$ conservation at the interface and/or the fact that the constant energy surfaces are not centered at $k = 0$ in the insulator, would lead to a more widely spread distribution. Anyway, tunneling in monocrystalline Pb[1] exhibits two gap

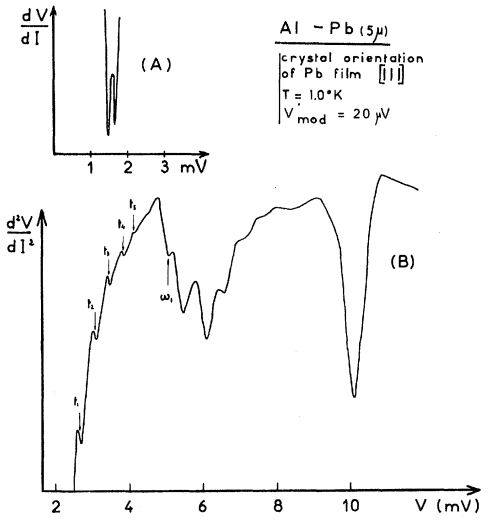


FIG. 1. (a) $dV/dI$ tunnel characteristic for thick Pb exhibiting a double gap edge. (b) $d^2V/dI^2$ curve for the same sample; notice the nonsplit phonon peaks and the Tomasch resonances ($t_n$).

thresholds almost independent of orientation, and, within the framework of Ref. 4, this could be experimental evidence for a rather isotropic distribution function $g(\Omega)$. For simplicity, we shall make this assumption later on. The complex gap function is given by

$$\Delta(\Omega, E) = \Delta_R(\Omega, E) + i\Delta_I(\Omega, E) .$$

To obtain the gap threshold of $N_T(E)$, Bennett needs to know $\Delta(\Omega, E)$ for small values of $E$, which he approximates by $\Delta_R(\Omega, 0)$. Then if $\Delta_R(\Omega, 0)$ depends on $\Omega$ and has two stationary values $\Delta_1$ and $\Delta_2$, $N_T(E)$ will have two singularities for $E = \Delta_1$ and $\Delta_2$.

To obtain phonon effects in $N_T(E)$, one considers relation (1) for $E \gg \Delta$,

$$\frac{N_T(E)}{N_0} \simeq \int d\Omega g(\Omega) \left(1 + \frac{\Delta_R^2(\Omega, E) - \Delta_I^2(\Omega, E)}{2E^2}\right) .$$

Near phonon critical points, the real part of the gap, $\Delta_R$, is almost constant, but the imaginary part $\Delta_I$ increases rapidly, and this gives a structure in $N_T(E)$.[5]

These two problems can readily be seen to be different. In the gap-edge case, the value of the real part, $\Delta_R$, is needed and in the phonon-structure case, it is the variation of the imaginary part, $\Delta_I$, which is important. Their anisotropies therefore may be quite different. Let us consider the last case. The energy gap is given by[6]

$$\Delta(\vec{p}, E) = \frac{N_0}{Z(\vec{p}, E)} \int d^2\vec{p}'\, dE'\, d\omega \times \mathrm{Re}\left(\frac{\Delta(\vec{p}', E')}{[E'^2 - \Delta^2(\vec{p}', E')]^{1/2}}\right) \times \left[\left(\frac{1}{E' + E + \omega - i\delta} + \frac{1}{E' - E + \omega - i\delta}\right) \times \sum_\lambda |g_{\lambda, \vec{p}'-\vec{p}}|^2 \delta(\omega - \omega_{\lambda, \vec{p}'-\vec{p}}) - U_0\right] ,$$

where $\vec{p}$ is the electron momentum (in direction $\Omega$), $N_0$ the Fermi density of states, $Z(\vec{p}, E)$ the renormalization factor, $\omega_{\lambda, \vec{q}}$ the dispersion relation for phonons, $g_{\lambda, \vec{q}}$ the electron-phonon interaction, and $U_0$ the Coulomb pseudopotential. The integration is performed over all electron momenta $\vec{p}'$ on the Fermi surface, the phonon wave vector being $\vec{p}' - \vec{p}$ [Fig. 2(a)].

The imaginary part of $\Delta(\vec{p}, E)$ is needed, and its main features can be retained by simply taking $Z(\vec{p}, E)$ to be constant and real[5] [this rather crude approximation being adequate for our purpose as it is analogous to assume that the $d^2I/dV^2$ characteristic gives $\alpha^2F(\omega - \Delta)$], which leads to a straightforward $\omega$ integration. In the sum, the important contribution comes from $E' \sim \Delta(\vec{p}')$. The $\vec{p}'$ dependence of $\Delta(\vec{p}')$ is known from Bennett's results. The integral is mainly sensitive to the stationary values of $\Delta(\vec{p}')$, so we shall assume that $\Delta(\vec{p}')$ just oscillates rapidly with $\vec{p}'$ between two values $\Delta_1$ and $\Delta_2$ (with the same weight). Then the $\vec{p}'$ integration leads to

$$\Delta_I(\Omega, E) = \frac{N}{Z} \int dE' \, \frac{1}{2}\left[\mathrm{Re}\left(\frac{\Delta_1}{[E'^2 - \Delta_1^2]^{1/2}}\right) \times \alpha^2F_\Omega(E - E') + (\Delta_1 \to \Delta_2)\right] , \qquad (2)$$

where

$$\alpha^2F_\Omega(\omega) = \sum_\lambda \int d^2\vec{p}' \, |g_{\lambda, \vec{p}-\vec{p}'}|^2 \delta(\omega - \omega_{\lambda, \vec{p}-\vec{p}'})$$

is the phonon spectrum involved. The angular dependence of this phonon density of states may be written[4]

$$F_\Omega(\omega) = F_0(\omega) + K_1(\theta, \phi)F_1(\omega) + K_2(\theta, \phi)F_2(\omega) .$$

$K_1(\theta, \phi)$ and $K_2(\theta, \phi)$ are cubic harmonics, $F_0(\omega)$ is the isotropic part, and $F_1(\omega)$ and $F_2(\omega)$ the anisotropic parts of this density of states.

Taking into account this expression of $F_\Omega(E)$, the phonon density of states can be plotted for each direction $\Omega = (\theta, \phi)$. The variation of $F_\Omega(E)$ with $\Omega$ results in a change of the line shape and a shift of the phonon peaks. If we estimate the location in energy of the longitudinal phonon by the first-order momentum of the corresponding peak, we find a shift of 310 $\mu$eV between the [111] and [001] directions.

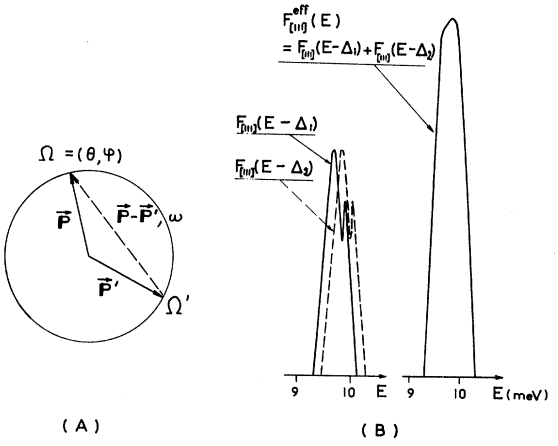


FIG. 2. (a) Free-electron Fermi sphere with electron and phonon momenta involved in the Eliashberg's equations. (b) Tunneling phonon density of states $F^{\mathrm{eff}}(E)$ involved in multigap Pb. $F^{\mathrm{eff}}$ is the sum of two curves shifted by $\Delta_2 - \Delta_1$; however, the intrinsic width of the peaks prevents the splitting from being resolved in the sum.

Changing the integration variable in Eq. (2), we obtain

$$\Delta_I(\Omega, E) \simeq \frac{N\alpha^2}{Z} \int \mathrm{Re} \frac{\Delta}{[(\Delta+x)^2-\Delta^2]^{1/2}} \times \tfrac{1}{2}[F_\Omega(E-\Delta_1-x)+F_\Omega(E-\Delta_2-x)]dx \ . \quad (3)$$

This expression is the same as that used by Scalapino, Schrieffer, and Wilkins[7] in order to explain the phonon structure in lead, but with an effective phonon density of states given by

$$F_\Omega^{\mathrm{eff}}(E) = \tfrac{1}{2}[F_\Omega(E-\Delta_1-x)+F_\Omega(E-\Delta_2-x)] \ .$$

If one considers the longitudinal peak region, for each value of $x$ and $\Omega$, the two functions $F$ are maxima at $E=\hbar\omega_{ph}+\Delta_{1,2}+x$. In Fig. 2(b) we have plotted, from Bennett's data for the [111] direction, $F_{[111]}(E-\Delta_1)$, $F_{[111]}(E-\Delta_2)$, and $F^{\mathrm{eff}}_{[111]}(E)$. The natural width of the peaks has already washed out most of the multigap effect on $F^{\mathrm{eff}}_{[111]}$.

The mathematical operations relating $F^{\mathrm{eff}}$ to the tunneling density of states are a convolution (3) and an angular average (1), the latter yielding a very strong broadening effect, as it mixes curves which are shifted with respect to one another (by 310 $\mu$eV for the [111] and [001] directions, for example). This shift must be compared to $|\Delta_2-\Delta_1| = 150$ $\mu$eV. Therefore, clearly, a simple phonon peak is expected.

In our experiment, we have used two kinds of Al-Pb junctions. Thin lead films (1000 Å) with short mean free paths give a single gap. Thick annealed lead films (1–8 $\mu$) show a double gap. In each case, the second derivative of the tunneling current has been recorded, and the phonon structure is essentially the same. Especially, as we have asserted at the beginning, in all cases there is no splitting in the phonon structure.

In conclusion, we believe to have shown that there is no splitting in the phonon spectrum due to multigap in a tunneling experiment, in agreement with all the experimental results to date.

We thank M. Belin for his technical assistance and Professor J. Bok for many valuable discussions.

*Work supported in part by Direction des Recherches et Moyens d'Essais, Contract No. 70/147.

[1]B. L. Blackford and R. H. March, Phys. Rev. 186, 397 (1969).

[2]J. M. Rowell and L. Kopf, Phys. Rev. 137, A907 (1965).

[3]W. J. Tomasch, in *Tunneling Phenomena in Solids*, edited by E. Burstein and S. Lundqvist (Plenum, New York, 1968), p. 316.

[4]A. J. Bennett, Phys. Rev. 140, A1902 (1965).

[5]D. J. Scalapino, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969).

[6]G. M. Eliashberg, Zh. Eksperim. i Teor. Fiz. 38, 966 (1960) [Sov. Phys. JETP 11, 696 (1960)].

[7]D. J. Scalapino, J. R. Schrieffer, and J. W. Wilkins, Phys. Rev. 148, 263 (1966).

PHYSICAL REVIEW B VOLUME 3, NUMBER 11 1 JUNE 1971

## ERRATUM

**Local Magnetic Fields in the Vanadium-Manganese Alloy System**, E. von Meerwall and D. S. Schreiber [Phys. Rev. B 3, 1 (1971)]. The affiliation of the first author should read: Department of Metallurgy and Mining Engineering, *and Materials Research Laboratory*, University of Illinois, Urbana.

On p. 3, first column, the sentence beginning in line 9 of Sec. III should be: "No further distortion is observed above 15% Mn, where about 0.8 of the peak-to-peak intensity per V nucleus which is present in pure V resides in a region somewhat outside the peak-to-peak region of the differential spectrum."

On p. 7, second column, the second sentence in the new paragraph should refer not to a *model* shift, but to the *modal* (most probable) shift.

On p. 20, Appendix C, in the next-to-last equation, a set of parentheses was inadvertently omitted from a denominator $(Z-5)$.